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POSITIVE COLUMN OF AN ELECTRIC ARC WITH A GIVEN POWER DENSITY DISTRIBUTION OF INTERNAL HEAT SOURCES

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An analytical solution is obtained for the nonlinear boundary-value problem describing heating of gases in the positive column of an arc with current and gas consumption varying along a channel for a given power density distribution of the internal heat sources.

To assure optimal reaction conditions in plasmachemical reactors it is often necessary to distribute the power density of the internal heat sources in a definite manner along the channel length. An analogous situation holds in processes for treating different powders and materials in an arc plasma. The most natural way to solve this problem is to change the current intensity, the size of the arc chamber, and the gas consumption along the length of the positive column. Such plasmatrons can be called electric arc heaters with distributed parameters, which are of interest both from the viewpoint of raising the plasmatron resources, and from the optimal distribution of heat fluxes [1, 2].

The power density distribution of the heat sources can be given, in many cases of practical importance, by the conditions of the technological process. However, the current distribution would be unknown in advance; hence, there is a necessity to solve the problem for a given power density distribution. Such a problem is also urgent from the viewpoint of designing plasmachemical reactors and a number of other electric arc devices.

The system of equations

$$\frac{h_s\rho u}{l}\frac{\partial S}{\partial z}+\frac{h_s\rho v}{R}\frac{\partial S}{\partial r}=\frac{1}{R^2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right)+\sigma_s E^2 S-\varepsilon_s S, \tag{1}$$

$$\frac{1}{l}\frac{\partial}{\partial z}(\rho u)+\frac{1}{R}\frac{1}{r}\frac{\partial}{\partial r}(r\rho v)=0, \tag{2}$$

$$I(z)=2\pi R^2\sigma_s E(z)\int_0^{\zeta} Srdr, \tag{3}$$

describing the heating of gases in a positive arc column with gas consumption and current distributed along the channel length [3, 4], is solved under the boundary conditions

$$S(r,\ 0)=\varphi(r/\zeta),\quad S_r(0,\ z)=0,\quad S(\zeta,\ z)=0,\quad v(0,\ z)=0. \tag{4}$$

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Let us consider the case of a linear change in the gas consumption across the arc section

$$\rho u = \frac{G_0}{\pi R^2}\,\Psi(z), \quad \Psi(z) = \frac{\alpha(z)}{\zeta^2(z)}, \quad \alpha(z) = 1 + klz. \tag{5}$$

Taking account of (2) under the conditions (4) and (5), Eq. (1) is reduced to the form

$$\Psi\,\frac{\partial S}{\partial z} - \frac{\Psi_z}{2}\,r\,\frac{\partial S}{\partial r} = \frac{a^2}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial S}{\partial r}\right) + cE^2S - bS, \tag{6}$$

$$a^2 = \pi l/G_0 h_s, \quad c = a^2R^2\sigma_s, \quad b = a^2R^2\varepsilon_s.$$

Substituting the expression for the electric field intensity from (3) into (6) results in a nonlinear integrodifferential equation whose solution is accompanied by significant difficulties. However, the insertion of a function U defined by the relationship

$$S(r, z) = U(r, z)\exp\left(\int\limits_0^z \frac{cE^2 - b}{\Psi}\,dz\right) \tag{7}$$

permits obtaining the linear equation

$$\Psi\,\frac{\partial U}{\partial z} = \frac{\Psi_z}{2}\,r\,\frac{\partial U}{\partial r} + \frac{a^2}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial U}{\partial r}\right)$$

from (6), which is reduced by the change of variable $x = z$, $\eta = r^2/\zeta^2(z)$ to the form

$$\alpha(x)\,\frac{\partial U}{\partial x} = 4a^2\,\frac{\partial}{\partial \eta}\left(\eta\,\frac{\partial U}{\partial \eta}\right) + kl\eta\,\frac{\partial U}{\partial \eta}. \tag{8}$$

Let us apply the integral transform

$$\bar{U}(\gamma, x) = \int\limits_0^1 U(\eta, x)\,P(\eta)\,K(\eta, \gamma)\,d\eta \tag{9}$$

to (8). Here $P(\eta)$ is a weight function reducing the differential operator of the right side of (8) to the self-adjoint form

$$\left[4a^2\,\frac{\partial}{\partial \eta}\left(\eta\,\frac{\partial U}{\partial \eta}\right) + kl\eta\,\frac{\partial U}{\partial \eta}\right] P(\eta) \equiv \frac{\partial}{\partial \eta}\left[g(\eta)\,\frac{\partial U}{\partial \eta}\right].$$

In our case

$$P(\eta) = \exp\left(\frac{kl}{4a^2}\,\eta\right), \quad g(\eta) = 4a^2\eta P(\eta).$$

The integral transform of the right side of (8) yields

$$\int\limits_0^1 \frac{\partial}{\partial \eta}\left[4a^2\eta\exp\left(\frac{kl}{4a^2}\,\eta\right)\frac{\partial U}{\partial \eta}\right] K(\eta, \gamma)\,d\eta = 4a^2\eta\exp\left(\frac{kl}{4a^2}\,\eta\right) \times$$
$$\times\left[\frac{\partial U}{\partial \eta}\,K - U\,\frac{\partial K}{\partial \eta}\right]\Bigg|_{\eta=0}^{\eta=1} + \int\limits_0^1 \frac{\partial}{\partial \eta}\left[4a^2\eta\exp\left(\frac{kl}{4a^2}\,\eta\right)\frac{\partial K}{\partial \eta}\right] U d\eta. \tag{10}$$

The transformed equation will not contain integral terms if

$$\frac{\partial}{\partial \eta}\left[4a^2\eta\exp\left(\frac{kl}{4a^2}\,\eta\right)\frac{\partial K}{\partial \eta}\right] = -a^2\nu^2 K(\eta, \gamma)\,P(\eta), \tag{11}$$

where ν^2 is a quantity independent of η. Taking account of (9) and (11) we find

$$\int\limits_0^1 \frac{\partial}{\partial \eta}\left[4a^2\eta\exp\left(\frac{kl}{4a^2}\,\eta\right)\frac{\partial K}{\partial \eta}\right] U d\eta = -a^2\nu^2\bar{U}(\gamma, x).$$

Using (4), the terms outside the integral in (10) can vanish if compliance with the conditions

$$K(1, \gamma) = 0, \quad K(0, \gamma) = \infty \tag{12}$$

is demanded. We take the relationship (11) as a differential equation governing the kernel of the integral transform. Replacement by the variable $\tau = -kl\eta/4a^2$ reduces (11) to the differential equation of a degenerate hypergeometric function

$$\tau \frac{\partial^2 K}{\partial \tau^2} + (1-\tau)\frac{\partial K}{\partial \tau} - \frac{\nu^2 a^2}{kl} K = 0. \tag{13}$$

Therefore, the degenerate hypergeometric function

$$K_n(\eta) = \Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\eta\right) \Big/ \|\Phi_n\| \tag{14}$$

is the solution of (11), where ν_n^2 are the eigenvalues of (13) assuring satisfaction of the boundary conditions (12), and $K_n(\eta)$ are the eigenfunctions corresponding to the eigenvalues of the problem (11), (12). The eigenvalues are determined from the solution of the equation

$$\Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\right) = 0,$$

where $\beta = kl/a^2$ is a dimensionless blowing parameter.

The eigenfunctions $K_n(\eta)$ corresponding to different eigennumbers are pairwise orthogonal with weight $P(\eta)$ in the interval $0 \le \eta \le 1$. It hence follows that eigenfunctions normalized with the same weight satisfy the condition

$$\int_0^1 K_i(\eta) K_j(\eta) P(\eta)\, d\eta = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases}$$

We take the orthonormalized eigenfunctions $K_n(\eta)$ as the kernel $K(\eta, \gamma)$. Then the integral transform of the left side of (8) yields

$$\int_0^1 \alpha(x) \frac{\partial U}{\partial x} P(\eta) K_n(\eta)\, d\eta = \alpha(x) \frac{d}{dx} \bar{U}_n(x).$$

Combining the results of the integral transforms, we obtain the differential equation in the transforms

$$\alpha(x) \frac{d\bar{U}_n(x)}{dx} = -a^2\nu_n^2 \bar{U}_n(x),$$

whose solution will be

$$\bar{U}_n(x) = \bar{U}_n(0) \exp\left(-\int_0^x \frac{a^2\nu_n^2}{\alpha(x)}\, dx\right). \tag{15}$$

If the function $U(\eta, x)$ is such that the integral

$$\int_0^1 U^2(\eta, x) P(\eta)\, d\eta$$

exists and is bounded uniformly relative to the set of values that can be taken by the parameters β and x, then the function $U(\eta, x)$ can be represented in the form of the series

$$U(\eta, x) = \sum_{n=1}^{\infty} U_n(x) K_n(\eta), \tag{16}$$

$$U_n(x) = \int_0^1 U(\eta, x) K_n(\eta) P(\eta)\, d\eta,$$

where the equality is understood in the sense of convergence in the mean [5]. From a comparison of (16) and the integral transform (9) for the desired function $U(\eta, x)$ it is seen that we should set $K(\eta, \gamma) = K_n(\eta)$. Here $K_n(\eta)$ are eigenfunctions of the problem (11) and (12). The integral transform of the function $U(\eta, x)$ agrees with its n-th coefficient of the series expansion in eigenfunctions of this problem [5]. The mutual one-to-one relationship between the function and its integral transform results from the uniqueness of the expansion (16). Therefore, the series (16) is a solution of (8) and the coefficients of the series $U_n(x)$ agree with (15). Substituting (14) and (15) into the series (16), we obtain

$$U(\eta, x) = \sum_{n=1}^{\infty} \frac{\bar{U}_n(0)}{\|\Phi_n\|} \exp\left(-\int_0^x \frac{a^2\nu_n^2}{\alpha(x)}\,dx\right) \Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\eta\right).$$

The coefficients $U_n(0)$ are determined from the boundary conditions in the initial section for $x = 0$, i.e.,

$$U(\eta, 0) = \varphi(\sqrt{\eta}) = \sum_{n=1}^{\infty} A_n \Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\eta\right),$$

$$A_n = \bar{U}_n(0)/\|\Phi_n\| = (\varphi(\sqrt{\eta}), \Phi_n)/(\Phi_n, \Phi_n),$$

$$(\varphi(\sqrt{\eta}), \Phi_n) = \int_0^1 \varphi(\sqrt{\eta})\,\Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\eta\right) P(\eta)\,d\eta,$$

$$\|\Phi_n\|^2 = (\Phi_n, \Phi_n) = \int_0^1 \Phi_n^2\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\eta\right) P(\eta)\,d\eta.$$

Making the transition from the variables x, η to z, r and taking account of (7), we find

$$S(r, z) = \exp\left(\int_0^z \frac{cE^2}{\Psi}\,dz\right) \exp\left(-\int_0^z \frac{b}{\Psi}\,dz\right) \sum_{n=1}^{\infty} A_n \exp\left(-\int_0^z \frac{a^2\nu_n^2}{\alpha(z)}\,dz\right) \Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4}\frac{r^2}{\zeta^2}\right). \qquad (17)$$

Multiplying (3) by E, we obtain an expression for the power density distribution of the heat sources

$$N(z) = 2\pi R^2 \sigma_s E^2 \int_0^{\zeta} S r\,dr. \qquad (18)$$

From (17) and (18) we find a nonlinear equation for the electric field intensity

$$N(z) = 2\pi R^2 \sigma_s E^2(z)\, F(z) \exp\left(\int_0^z \frac{cE^2}{\Psi}\,dz\right).$$

Here

$$F(z) = \exp\left(-\int_0^z \frac{b}{\Psi}\,dz\right) \sum_{n=1}^{\infty} \frac{\bar{U}_n(0)}{\|\Phi_n\|} \exp\left(-\int_0^z \frac{a^2\nu_n^2}{\alpha(z)}\,dz\right) \gamma_n \zeta^2,$$

$$\gamma_n = \int_0^1 \Phi_n\left(\frac{\nu_n^2}{\beta}, 1, -\frac{\beta}{4} r^2\right) r\,dr.$$

Let us represent it as follows:

$$\int_0^z \frac{c}{2\pi R^2 \sigma_s} \frac{N(z)\,dz}{F(z)\,\Psi(z)} = \int_0^z \frac{cE^2}{\Psi} \exp\left(\int_0^z \frac{cE^2}{\Psi}\,dz\right) dz,$$

from which it follows that

$$E(z) = \left[\frac{N(z)}{2\pi R^2 \sigma_s F(z)}\right]^{0.5} \left[1 + \frac{a^2}{2\pi} \int_0^z \frac{N(z)}{F(z)} \frac{dz}{\Psi(z)}\right]^{-0.5}. \qquad (19)$$

Substituting (19) into (17) yields, after simple manipulation, a formula to compute the distribution of the heat-conduction function

$$S(r,\ z)=\left[1+\frac{a^2}{2\pi}\int\limits_0^z\frac{N(z)}{F(z)}\frac{dz}{\Psi(z)}\right]\exp\left(-\int\limits_0^z\frac{b}{\Psi}dz\right)\times$$
$$\times\sum_{n=1}^{\infty}\frac{\bar{U}_n(0)}{\|\Phi_n\|}\exp\left(-\int\limits_0^z\frac{a^2\nu_n^2}{\alpha(z)}dz\right)\Phi_n\left(\frac{\nu_n^2}{\beta},\ 1,\ -\frac{\beta}{4}\ \frac{r^2}{\zeta^2}\right). \qquad (20)$$

The distribution of the current intensity along the channel length that corresponds to a given power distribution is defined as

$$I(z)=\left[2\pi R^2\sigma_s F(z)\,N(z)\left(1+\frac{a^2}{2\pi}\int\limits_0^z\frac{N(z)}{F(z)}\frac{dz}{\Psi}\right)\right]^{0,5}. \qquad (21)$$

The practical realization of the current distribution obtained is difficult technically. However, an approximate current distribution along the channel length can be obtained in a multielectrode plasmatron. A refined analysis of the given current distribution can be performed as follows. We obtain an integral equation for E from (3) and (18)

$$I(z)=2\pi R^2\sigma_s F(z)\,E(z)\exp\left(\int\limits_0^z\frac{cE^2}{\Psi}dz\right),$$

whose solution will be

$$E(z)=\frac{I(z)}{2\pi R^2\sigma_s F(z)}\left\{\int\limits_0^z\left[\frac{I(z)}{F(z)}\right]^2\frac{2cdz}{\Psi 4\pi^2R^4\sigma_s^2}+1\right\}^{-0,5}.$$

Eliminating E(z) from (17), we obtain a formula to compute the distribution of the heat-conduction function

$$S(r,\ z)=\left\{1+\int\limits_0^z\left[\frac{I(z)}{F(z)}\right]^2\frac{2c}{4\pi^2R^4\sigma_s^2}\ \frac{dz}{\Psi(z)}\right\}^{0,5}\times$$
$$\times\sum_{n=1}^{\infty}\frac{\bar{U}_n(0)}{\|\Phi_n\|}\exp\left(-\int\limits_0^z\frac{a^2\nu_n^2+b\zeta^2}{\alpha(z)}dz\right)\Phi_n\left(\frac{\nu_n^2}{\beta},\ 1,\ -\frac{\beta}{4}\ \frac{r^2}{\zeta^2}\right). \qquad (22)$$

The power per unit length of the positive column will be

$$N(z)=\frac{I^2(z)}{F(z)}\left\{\int\limits_0^z\left[\frac{I(z)}{F(z)}\right]^2\frac{2cdz}{\Psi(z)}+4\pi^2R^4\sigma_s^2\right\}^{-0,5}.$$

The heat fluxes through unit length of positive column arc are determined by the relationship

$$q=q_\varepsilon+q_\zeta=\frac{I(z)}{E(z)}\ \frac{\varepsilon_s}{\sigma_s}-2\pi\zeta\left.\frac{\partial S}{\partial r}\right|_{r=\zeta}.$$

The formulas obtained permit computation of the electrical and thermal characteristics of the column as a function of the gas properties and consumption, of the distributions of the gas consumption and the current, of the initial distribution of the heat-conduction function and the size of the arc chamber. The formulas obtained are also valid in those cases when σ_s and ε_s vary significantly over the length of the positive column.

NOTATION

I, E, electric field current and intensity; R, l, channel radius and length; r, z, cylindrical coordinates referred, respectively, to R and l; u, v, longitudinal and radial velocity components; G, gas mass flow rate per second through the column cross section; N(z), power per unit length of arc; σ, ρ, h, S_1, ε, electrical conductivity, density, enthalpy, heat-conduction function, and integrated volume radiation density; ζ, radius of the column referred to R; S_*, value of S_1 on the column boundary; $\sigma_S = \partial\sigma/\partial S$; $h_S = \partial h/\partial S$; $\varepsilon_S = \partial\varepsilon/\partial S$; $S = S_1 - S_*$; q_ζ, q_ε, energy losses per unit length of the positive column per unit time for the heat conduction and the radiation.

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THERMAL MODE OF A LAMP OPERATING IN THE PULSE MODE

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A method of computing the thermal modes of lamps is proposed and the passage to the analysis of thermooptical distortions occurring in their active elements is realized.

The effective conversion of pumping energy into heat causes optical inhomogeneity of the active elements. In turn, the optical inhomogeneity results in the appearance of a thermal lens, birefringence, divergence of the radiation. Hence, an analysis of these parameters is one of the fundamental problems occurring in the production of lamps.

In numerous papers devoted to the analysis of the thermal modes of lamps, the thermal state of just the active element is examined as a rule, without taking into account its relation to the other elements in the system [1-6]. At the same time, the thermal analyses do not permit a judgment about the quality of system operation. This is explained by the indirect influence of the thermal effects. Papers devoted to the investigation of thermooptical distortions in the active element are either based on a known temperature field, or are experimental in nature [7-10]. In this connection, the problem of developing thermal and mathematical models of lamps, the production of methods of analyzing their thermal mode, and the passage to an analysis of the thermooptical distortions in the active element is quite urgent.

This paper is devoted to the production of a method of analyzing the thermal mode of a lamp. The thermal mode of the active element is investigated in greatest detail, and the thermooptical distortions that occur are determined. The sequence presented below for the analysis is common to a broad class of constructions of lamps operating in the pulse mode.

Let us examine the example of analyzing the thermal mode of the lamp displayed schematically in Fig. 1. It consists of the following main elements: a cylindrical active element 1 fabricated from glass; two pumping lamps 2 operating in the single pulse mode; a reflecter 3, and a housing 4.

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